

## Analysis and Visualization of Classifier Performance: Comparison under Imprecise Class and Cost Distributions

**Foster Provost**

NYNEX Science and Technology  
400 Westchester Avenue  
White Plains, New York 10604  
foster@nynexst.com

**Tom Fawcett**

NYNEX Science and Technology  
400 Westchester Avenue  
White Plains, New York 10604  
fawcett@nynexst.com

### Abstract

Applications of inductive learning algorithms to real-world data mining problems have shown repeatedly that using accuracy to compare classifiers is not adequate because the underlying assumptions rarely hold. We present a method for the comparison of classifier performance that is robust to imprecise class distributions and misclassification costs. The ROC convex hull method combines techniques from ROC analysis, decision analysis and computational geometry, and adapts them to the particulars of analyzing learned classifiers. The method is efficient and incremental, minimizes the management of classifier performance data, and allows for clear visual comparisons and sensitivity analyses.

### Introduction

When mining data with inductive methods, we often experiment with a wide variety of learning algorithms, using different algorithm parameters, varying output threshold values, and using different training regimens. Such experimentation yields a large number of classifiers to be evaluated and compared. In order to compare the performance of classifiers it is necessary to know the conditions under which they will be used; using accuracy alone is inadequate because class distributions and misclassification costs are rarely uniform.

Decision-theoretic principles may be used if the class and cost distributions are known exactly. Unfortunately, on real-world problems target cost and class distributions can rarely be specified precisely, and they are often subject to change. For example, in fraud detection we cannot ignore either type of distribution, nor can we assume that our distribution specifications are static or precise. We need a method for the management and comparison of multiple classifiers that is robust to imprecise and changing environments.

We introduce the *ROC convex hull method*, which combines techniques from ROC analysis, decision anal-

ysis and computational geometry. The method decouples classifier performance from specific class and cost distributions, and may be used to specify the subset of methods that are potentially optimal under any cost and class distribution assumptions.

The ROC convex hull method is efficient, so it facilitates the comparison of a large number of classifiers. It minimizes the management of classifier performance data, because it can specify exactly those classifiers that are potentially optimal, and it is incremental, easily incorporating new and varied classifiers.

### The Inadequacy of Accuracy

A tacit assumption in the use of classification accuracy as an evaluation metric is that the class distribution among examples is *constant and relatively balanced*. In the real world this is rarely the case. Classifiers are often used to sift through a large population of normal or uninteresting entities in order to find a relatively small number of unusual ones, for example, looking for defrauded customers or checking an assembly line for defective parts. Because the unusual or interesting class is rare among the general population, the class distribution is very skewed (Ezawa, Singh, & Norton 1996; Fawcett & Provost 1996).

As the class distribution becomes more skewed, evaluation based on accuracy breaks down. Consider a domain where the classes appear in a 999:1 ratio. A simple rule, always classify as the maximum likelihood class, gives a 99.9% accuracy. Presumably this is not satisfactory if a non-trivial solution is sought. Skews of  $10^2$  are common in fraud detection and skews greater than  $10^6$  have been reported in other classifier learning applications (Clearwater & Stern 1991).

Evaluation by classification accuracy also tacitly assumes *equal error costs*—that a false positive error is equivalent to a false negative error. In the real world this is rarely the case, because classifications lead to actions which have consequences, sometimes grave. Rarely are mistakes evenly weighted in their cost. Indeed, it is hard to imagine a domain in which a learning system may be indifferent to whether it makes a false positive or a false negative error. In such cases,

accuracy maximization should be replaced with cost minimization.

The problems of unequal error costs and uneven class distributions are related. It has been suggested that high-cost instances can be compensated for by increasing their prevalence in an instance set (Breiman *et al.* 1984). Unfortunately, little work has been published on either problem. There exist several dozen articles (Turney 1996) in which techniques are suggested, but little is done to evaluate and compare them (the article of Pazzani *et al.* (1994) being the exception). The literature provides even less guidance in situations where distributions are imprecise or can change.

## Evaluating and Visualizing Classifier Performance

To discuss classifier evaluation we use the following terminology. Let  $\{\mathbf{p}, \mathbf{n}\}$  be the positive and negative instance classes, and let  $\{\mathbf{Y}, \mathbf{N}\}$  be the classifications produced by a classifier. Let  $p(\mathbf{p}|I)$  be the posterior probability that instance  $I$  is positive. The true positive rate,  $TP$ , of a classifier is:

$$TP = p(\mathbf{Y}|\mathbf{p}) \approx \frac{\text{positives correctly classified}}{\text{total positives}}$$

The false positive rate,  $FP$ , of a classifier is:

$$FP = p(\mathbf{Y}|\mathbf{n}) \approx \frac{\text{negatives incorrectly classified}}{\text{total negatives}}$$

Let  $c(\text{classification}, \text{class})$  be a two-place error cost function where  $c(\mathbf{Y}, \mathbf{n})$  is the cost of a false positive error and  $c(\mathbf{N}, \mathbf{p})$  is the cost of a false negative error<sup>1</sup>. If a classifier produces posterior probabilities, decision analysis gives us a way to produce cost-sensitive classifications from the classifier (Weinstein & Fineberg 1980). Classifier error frequencies can be used to approximate probabilities (Pazzani *et al.* 1994). For an instance  $I$ , the decision to emit a positive classification is:

$$[1 - p(\mathbf{p}|I)] \cdot c(\mathbf{Y}, \mathbf{n}) < p(\mathbf{p}|I) \cdot c(\mathbf{N}, \mathbf{p})$$

Regardless of whether a classifier produces probabilistic or binary classifications, its normalized cost on a test set can be evaluated empirically as:

$$\text{Cost} = FP \cdot c(\mathbf{Y}, \mathbf{n}) + FN \cdot c(\mathbf{N}, \mathbf{p})$$

Given a set of classifiers, a set of examples, and a precise cost function, most work on cost-sensitive classification uses an equation such as this to rank the classifiers according to cost and chooses the minimum. However, as discussed above, such analyses assume that the distributions are precise and static.

Receiver Operating Characteristic (ROC) graphs have long been used in signal detection theory to depict tradeoffs between hit rate and false alarm rate

<sup>1</sup>Error costs include benefits not realized.

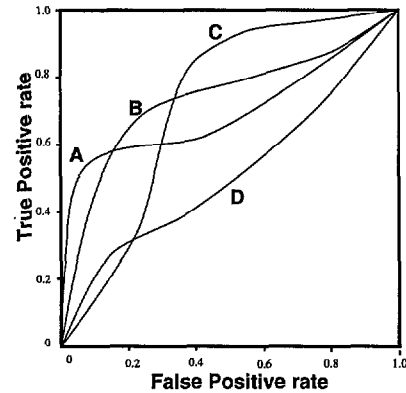


Figure 1: An ROC graph of four classifiers

(Egan 1975). ROC analysis has been extended for use in visualizing and analyzing the behavior of diagnostic systems (Swets 1988), and is used for visualization in medicine (Beck & Schultz 1986).

We will use the term *ROC space* to denote the classifier performance space used for visualization in ROC analysis. On an ROC graph,  $TP$  is plotted on the Y axis and  $FP$  is plotted on the X axis. These statistics vary together as a threshold on a classifier's continuous output is varied between its extremes, and the resulting curve is called the ROC curve. An ROC curve illustrates the error tradeoffs available with a given classifier. Figure 1 shows a plot of the performance of four classifiers, A through D, typical of what we see in the creation of alarms for fraud detection (Fawcett & Provost 1996).

For orientation, several points on an ROC graph should be noted. The lower left point (0,0) represents the strategy of never alarming, the upper right point (1,1) represents the strategy of always alarming, the point (0,1) represents perfect classification, and the line  $y = x$  (not shown) represents the strategy of randomly guessing the class. Informally, one point in ROC space is better than another if it is to the northwest ( $TP$  is higher,  $FP$  is lower, or both). An ROC graph allows an informal visual comparison of a set of classifiers. In Figure 1, curve A is better than curve D because it dominates in all points.

ROC graphs illustrate the behavior of a classifier *without regard to class distribution or error cost*, and so they decouple classification performance from these factors. Unfortunately, while an ROC graph is a valuable visualization technique, ROC analysis does a poor job of aiding the choice of classifiers. Only when one classifier clearly dominates another over the entire performance space can it be declared better. Consider the classifiers shown in Figure 1. Which is best? The answer depends upon the performance requirements, *i.e.*, the error costs and class distributions in effect when the classifiers are to be used.

Some researchers advocate choosing the classifier

that maximizes the product  $(1 - FP) \cdot TP$ . Geometrically, this corresponds to fitting rectangles under every ROC curve and choosing the rectangle of greatest area. This and other approaches that calculate average performance over the entire performance space (Swets 1988; Beck & Schultz 1986) may be appropriate if costs and class distributions are completely unknown and a single classifier must be chosen to handle any situation. However, they will choose a suboptimal classifier in many situations.

### The ROC Convex Hull Method

In this section we combine decision analysis with ROC analysis and adapt them for comparing the performance of a set of learned classifiers. The method is based on three high-level principles. First, the ROC space is used to separate classification performance from class and cost distribution information. Second, decision-analytic information is projected onto the ROC space. Third, we use the convex hull in ROC space to identify the subset of methods that are potentially optimal.

#### Iso-performance lines

By separating classification performance from class and cost distribution assumptions, the decision goal can be projected onto ROC space for a neat visualization. Formally, let the prior probability of a positive example be  $p(p)$ , so the prior probability of a negative example is  $p(n) = 1 - p(p)$ . Costs of false positive and false negative errors are given by  $c(Y, n)$  and  $c(N, p)$ , respectively. The expected cost of a classification by the classifier represented by a point  $(TP, FP)$  in ROC space is:

$$p(p) \cdot (1 - TP) \cdot c(N, p) + p(n) \cdot FP \cdot c(Y, n)$$

Therefore, two points,  $(TP_1, FP_1)$  and  $(TP_2, FP_2)$ , have the same performance if

$$\frac{TP_2 - TP_1}{FP_2 - FP_1} = \frac{p(n)c(Y, n)}{p(p)c(N, p)}$$

This equation defines the slope of an *iso-performance line*, *i.e.*, all classifiers corresponding to points on the line have the same expected cost. Each set of class and cost distributions defines a family of iso-performance lines. Lines “more northwest”—having a larger  $TP$ -intercept—are better because they correspond to classifiers with lower expected cost.

#### The ROC convex hull

Because in most real-world cases the target distributions are not known precisely, it is valuable to be able to identify what subset of classifiers is potentially optimal. Each possible set of distributions defines a family of iso-performance lines, and for a given family, the optimal methods are those that lie on the “most-northwest” iso-performance line. Thus, a classifier is

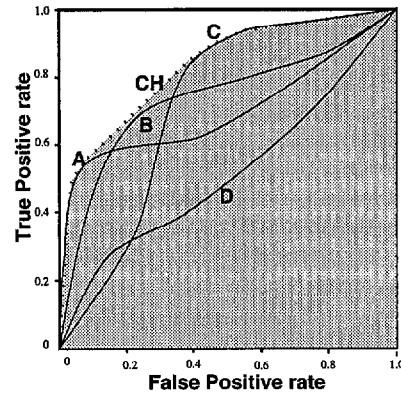


Figure 2: The ROC convex hull identifies potentially optimal classifiers.

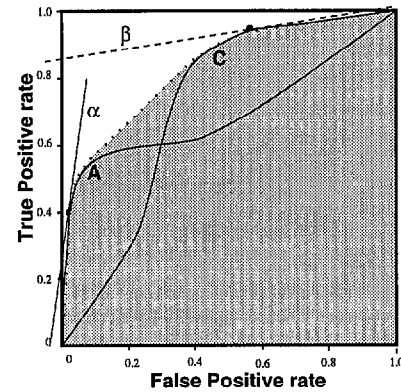


Figure 3: Lines  $\alpha$  and  $\beta$  show the optimal classifier under different sets of conditions.

potentially optimal if and only if it lies on the northwest boundary (*i.e.*, above the line  $y = x$ ) of the convex hull (Barber, Dobkin, & Huhdanpaa 1993) of the set of points in ROC space. Space limitations prohibit a formal proof, but one can see that *if a point lies on the convex hull*, then there exists a line through that point such that no other line with the same slope through any other point has a larger  $TP$ -intercept, and thus *the classifier represented by the point is optimal* under any distribution assumptions corresponding to that slope. *If a point does not lie on the convex hull*, then for any family of iso-performance lines there is another point that lies on an iso-performance line with the same slope but larger  $TP$ -intercept, and thus *the classifier cannot be optimal*.

We call the convex hull of the set of points in ROC space the *ROC convex hull* of the corresponding set of classifiers. Figure 2 shows the curves of Figure 1 with the ROC convex hull drawn (CH, the border between the shaded and unshaded areas). D is clearly not optimal. Surprisingly, B can never be optimal either because none of the points of its ROC curve lies on the convex hull. We can also remove from consideration

any points of A and C that do not lie on the hull.

Consider these classifiers under two distribution scenarios. In each, negative examples outnumber positives by 10:1. In scenario  $\mathcal{A}$ , false positive and false negative errors have equal cost. In scenario  $\mathcal{B}$ , a false negative is 100 times as expensive as a false positive (e.g., missing a case of fraud is much worse than a false alarm). Each scenario defines a family of iso-performance lines. The lines corresponding to scenario  $\mathcal{A}$  have slope 10; those for  $\mathcal{B}$  have slope  $\frac{1}{10}$ . Figure 3 shows the convex hull and two iso-performance lines,  $\alpha$  and  $\beta$ . Line  $\alpha$  is the “best” line with slope 10 that intersects the convex hull; line  $\beta$  is the best line with slope  $\frac{1}{10}$  that intersects the convex hull. Each line identifies the optimal classifier under the given distribution.

## Generating the ROC Convex Hull

We call the comparison of classifier performance based on the ROC convex hull and iso-performance lines the *ROC convex hull method*.

1. For each classifier, plot  $TP$  and  $FP$  in ROC space. For continuous-output classifiers, vary a threshold over the output range and plot the ROC curve.
2. Find the convex hull of the set of points representing the predictive behavior of all classifiers of interest. For  $n$  classifiers this can be done in  $O(n \log(n))$  time by the QuickHull algorithm (Barber, Dobkin, & Huhdanpaa 1993).
3. For each set of class and cost distributions of interest, find the slope (or range of slopes) of the corresponding iso-performance lines.
4. For each set of class and cost distributions, the optimal classifier will be the point on the convex hull that intersects the iso-performance line with largest  $TP$ -intercept. Ranges of slopes specify hull segments.

## Using the ROC Convex Hull

Figures 2 and 3 demonstrate how the subset of classifiers that are potentially optimal can be identified and how classifiers can be compared under different cost and class distributions. We now demonstrate additional benefits of the method.

## Comparing a variety of classifiers

The ROC convex hull method accommodates both binary and continuous classifiers. Binary classifiers are represented by individual points in ROC space. Continuous classifiers produce numeric outputs to which thresholds can be applied, yielding a series of  $(FP, TP)$  pairs comprising an ROC curve. Each point may or may not contribute to the ROC convex hull. Figure 4 depicts the binary classifiers E, F and G added to the previous hull. E may be optimal under some circumstances because it extends the convex hull. Classifiers F and G will never be because they do not extend it.

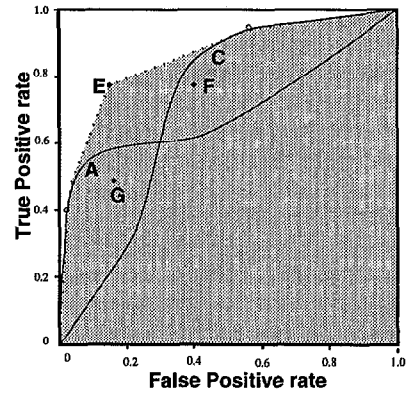


Figure 4: Classifier E may be optimal because it extends the ROC convex hull. F and G cannot because they do not.

New classifiers can be added incrementally to an ROC convex hull analysis, as demonstrated above with the addition of classifiers E, F, and G. Each new classifier either extends the existing hull or does not. In the former case the hull must be updated accordingly, but in the latter case the new classifier can be ignored. Therefore, the method does not require saving every classifier (or saving statistics on every classifier) for re-analysis under different conditions—only those points on the convex hull. No other classifiers can ever be optimal, so they need not be saved. Every classifier that *does* lie on the convex hull must be saved.

## Changing distributions and costs

Class and cost distributions that change over time necessitate the reevaluation of classifier choice. In fraud detection, costs change based on workforce and reimbursement issues; the amount of fraud changes monthly. With the ROC convex hull method, comparing under a new distribution involves only calculating the slope(s) of the corresponding iso-performance lines and intersecting them with the hull, as shown in Figure 3.

The ROC convex hull method scales gracefully to any degree of precision in specifying the cost and class distributions. If nothing is known about a distribution, the ROC convex hull shows all classifiers that may be optimal under any conditions. Figure 2 showed that, given classifiers A, B, C and D of Figure 1, only A and C can ever be optimal.

With complete information, the method identifies the optimal classifier(s). In Figure 3 we saw that classifier A (with a particular threshold value) is optimal under scenario  $\mathcal{A}$  and classifier C is optimal under scenario  $\mathcal{B}$ . Next we will see that with less precise information, the ROC convex hull can show the set of possibly optimal classifiers.

## Sensitivity analysis

Imprecise distribution information defines a *range* of slopes for iso-performance lines. This range of slopes intersects a segment of the ROC convex hull, which facilitates sensitivity analysis. For example, if the segment defined by a range of slopes corresponds to a single point in ROC space or a small threshold range for a single classifier, then there is no sensitivity to the distribution assumptions in question. Consider a scenario similar to  $\mathcal{A}$  and  $\mathcal{B}$  in that negative examples are 10 times as prevalent as positive ones. In this scenario, the cost of dealing with a false alarm is between \$5 and \$10, and the cost of missing a positive example is between \$500 and \$1000. This defines a range of slopes for iso-performance lines:  $\frac{1}{20} \leq m \leq \frac{1}{5}$ . Figure 5a depicts this range of slopes and the corresponding segment of the ROC convex hull. The figure shows that the choice of classifier is insensitive to changes within this range (and tuning of the classifier's threshold will be relatively small). Figure 5b depicts a scenario with a wider range of slopes:  $\frac{1}{5} \leq m \leq 2$ . The figure shows that under this scenario the choice of classifier is very sensitive to the distribution. Classifiers A, C and E each are optimal for some subrange.

A particularly interesting question in any domain is, *When is a "do nothing" strategy better than any of my available classifiers?* Consider Figure 5c. The point (0, 0) corresponds to doing nothing, *i.e.*, issuing negative classifications regardless of input. Any set of cost and class distribution assumptions for which the best hull-intersecting iso-performance line passes through the origin (*e.g.*, line  $\alpha$ ) defines a scenario where this null strategy is optimal. In the example of Figure 5c, the range of scenarios is small for which the null strategy is optimal; the slopes of the lines quantify the range.

## Limitations and Implications

In this paper, we have simplified by assuming there are only two classes and that costs do not vary within a given type of error. The first assumption is essential to the use of a two dimensional graph; the second assumption is essential to the creation of iso-performance lines. Furthermore, the method is based upon the maximization of expected value as the decision goal. Other decision goals are possible (Egan 1975). For example, the *Neyman-Pearson observer* strategy tries to maximize the hit rate for a fixed false-alarm rate. In our framework, a Neyman-Pearson observer would find the vertical line corresponding to the given FP rate, and intersect it with a "non-decreasing" hull, rather than the convex hull (and move left horizontally, if possible). Also, methods such as these should consider statistical tests for comparing performance curves, so that the user has confidence that differences in performance are significant.

The tradeoff between TP and FP rates is similar to the tradeoff between precision and recall, com-

monly used in Information Retrieval (Bloedorn, Mani, & MacMillan 1996). However, precision and recall do not take into account the relative size of the population of "uninteresting" entities, which is necessary to deal with changing class distributions.<sup>2</sup>

Existing cost-sensitive learning methods are brittle with respect to imprecise or changing distributions. These methods can be categorized into four categories: (i) the use of cost distribution in building a classifier, *e.g.*, for choosing splits in a decision tree or for building rule sets (Breiman *et al.* 1984; Pazzani *et al.* 1994; Provost & Buchanan 1992); (ii) the use of the cost distribution in post-processing the classifier, *e.g.*, for pruning a decision tree (Breiman *et al.* 1984; Pazzani *et al.* 1994), for finding rule subsets (Catlett 1995; Provost & Buchanan 1995), or for setting an output threshold; (iii) estimate the probability distribution and use decision-analytic combination (Pazzani *et al.* 1994; Catlett 1995; Duda & Hart 1973); and (iv) search for a bias with which a good classifier can be learned (Turney 1995; Provost & Buchanan 1995). Of these, only probability estimation methods (iii) can handle changes in cost (or class) distribution without modifying the classifier. However, no single method dominates all others, so the ROC convex hull is still needed for comparison. As future work, we propose the development of methods that search explicitly for classifiers that extend the ROC convex hull.

## Conclusion

The ROC convex hull method is a robust, efficient solution to the problem of comparing multiple classifiers in imprecise and changing environments. It is intuitive, can compare classifiers both in general and under specific distribution assumptions, and provides crisp visualizations. It minimizes the management of classifier performance data, by selecting exactly those classifiers that are potentially optimal; thus, only these data need to be saved in preparation for changing conditions. Moreover, due to its incremental nature, new classifiers can be incorporated easily, *e.g.*, when trying a new parameter setting.

It has been noted many times that costs and class distributions are difficult to specify precisely. Classifier learning research should explore flexible systems that perform well under a range of conditions, perhaps for part of ROC space. We hope that our method for analysis of classifiers can help free researchers from the need to have precise class and cost distribution information.

<sup>2</sup>Thanks to Peter Turney for an enlightening discussion on the application of this approach to IR.

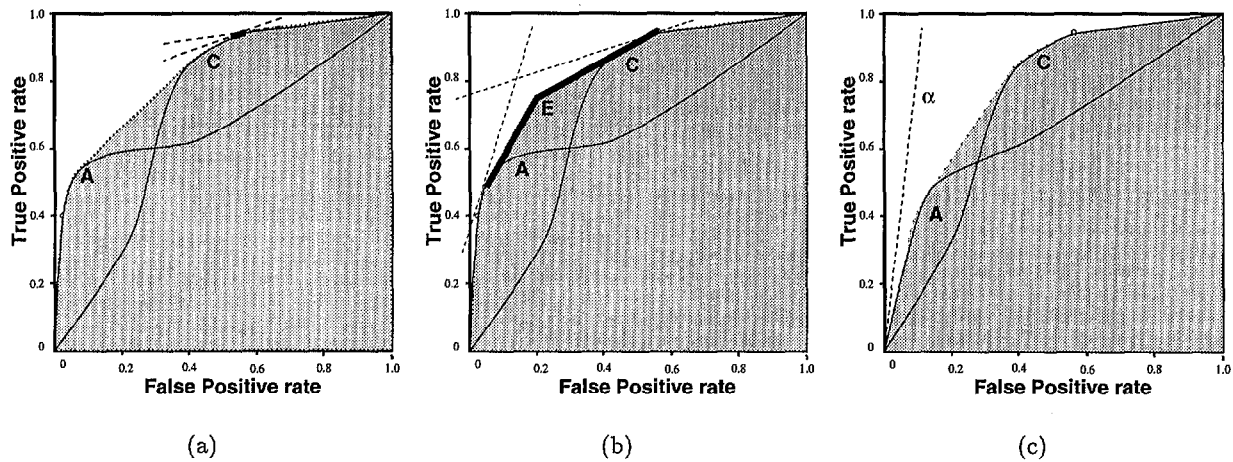


Figure 5: Sensitivity analysis using the ROC convex hull: (a) low sensitivity (only C can be optimal), (b) high sensitivity (A, E, or C can be optimal), (c) doing nothing is the optimal strategy

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